Compton scattered radiation imaging and Bimodality

Mai K. Nguyen
ETIS-CNRS 8051 / ENSEA/ Université de Cergy-Pontoise, France
mai.nguyen-verger@u-cergy.fr

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Contents

1) Principle of Compton Scattered Radiation Imaging (CSRI)

2) The CSRI modalities:
   - Emission scattering imaging
   - Transmission scattering imaging

3) A Bimodal Compton scattered radiation imaging process

4) Bibliography
1) Principle of Compton Scattered Radiation Imaging

**PRINCIPLE:**
The necessary information for reconstruction does not come from primary transmitted rays but from deflected rays by Compton scattering.

\[ E_{\omega} = \frac{E_0}{1 + \frac{E_0}{mc^2} (1 - \cos(\omega))} \]

Compton scattering of radiation (photons) by electrons
Principle of Scattered Radiation Imaging

- What is measured is the number of scattered photons of given energy $E_\omega$ at a given pixel site.

- This number is due to the contributions of many source sites (or scattering sites when the source is fixed) lying on surfaces in 3D and curves in 2D.

- As the detection pixel varies in space, the measurement process is modelled by generalized Radon transforms, which are integrals of activity density (or electron density) on surfaces (or curves).

- In scattered radiation emission imaging, the electron density is assumed to be known, the unknown quantity is the activity density.

- In transmission – scattering imaging, the external source is known and the unknown is the object electron density.

- Solving for the unknown density is an inverse problem, which requires the inversion of generalized Radon transforms.
2) Compton scattered radiation imaging modalities

A) Emission Imaging
(Radiating objects with source activity distribution):
- 3 D imaging (2002)
- 2 D imaging (2010)

B) Transmission Imaging
In two dimensions called Compton scattering Tomography (CST)
(Objects subjected to fixed external radiation source):
- A new modality (2010)

C) Bimodality:
A combination of emission and transmission scattering modalities
A review of conventional Emission Imaging: SPECT

- Data: Linear projections of activity density at energy $E_0$
- Modeling by X-ray transform of activity density along straight lines along all possible directions
- Reconstruction of activity by inversion of X-ray transform (existing)
Scattered Radiation SPECT imaging

- **Concept of compounded conical projection** of activity density at energy $E < E_0$
- **Compounded conical Radon transform** (CCRT) for all scattering angles (or energy $E$) and all detection sites $D$, electron density assumed known
Conventional detection of primary radiation by a collimated gamma camera

Detection of first order scattered radiation by gamma camera

Linear versus Conical projections
Detection of scattered radiation by collimated gamma camera

Contributions of scattered radiation at one detection site

Analysis of the photon detection on SPECT camera
Scattered Radiation Emission Imaging

- Object is characterized by activity density (radiating object)
- Detection pixel on collimated SPECT type γ-camera
- Flux density of detected scattered photons at site D is due to
  - all the scattering sites M on a line perpendicular to the γ-camera plane
  - all emission sites S on circular cones of apex opening angle \( \omega \) and the locus of all scattering sites as symmetry axis

For given electron density, the detected photon flux density at a site of the γ-camera is given by
  - an integral of the activity density over the surface of a circular cone and
  - an integral over the line of scattering sites

Such an integral, which represents the scattered radiation emission data is a function of three variables: the two coordinates of site D on the γ-camera and the photon scattered energy \( E \) (or the scattering angle \( \omega \)).

In two dimensions, the data is function of two variables: the abscissa of the detecting site on a linear γ-camera and \( \omega \) the scattering angle.
Image formation modelling by the Compounded Conical Radon transform of

object activity density \( f(x, y, z) \)

\[
\hat{f}(x_D, y_D, \omega) = \int_{\mathbb{R}^3} dx \, dy \, dz \, K_{PSF}(x_D, y_D, \omega|x, y, z) \, \bar{f}(x, y, z)
\]

where

\[
\bar{f}(x, y, z) = \int_0^\infty d\zeta \, \nu(\zeta) \, f(x, y, z + \zeta)
\]

integration kernel

\[
K_{PSF}(x_D, y_D, \omega|x, y, z)
\]

\[
= K(\omega) \, \nu(\sqrt{(x-x_D)^2 + (y-y_D)^2 + (z-\zeta)^2})
\]

\[
\times \delta(\cos \omega \sqrt{(x-x_D)^2 + (y-y_D)^2} - (z-\zeta) \sin \omega)
\]

\(K(\omega)\) is the Compton kinematic factor

\(\nu(d) = 1/d^2\) (photometric factor)
Reconstruction of activity by inversion of the Compounded Conical Radon Transform

The Fourier transform of $f(x, y, z)$ verifies

$$
\tilde{f}(u, v, w) = \int_{\mathbb{R}} d\sigma \exp[2i\pi\sigma w] \left[ -|z|\sqrt{u^2 + v^2} \right] \frac{\mathcal{J}(w)}{\mathcal{J}(w)} \int_{\mathbb{R}_+} t \, dt \, J_1(2\pi |z| t \sqrt{u^2 + v^2})
$$

$$
\left[ H(\omega - \pi/2) \frac{\partial}{\partial t} \frac{G(u, v, t)}{K(t)} + H(\pi/2 - \omega) \frac{\partial}{\partial t} \frac{G(u, v, -t)}{K(-t)} \right],
$$

where

- $\mathcal{J}(w)$ is the Fourier transform of $\nu(x)$,
- $J_1(x)$ is the Bessel function of order 1,
- $H(x)$ is the Heaviside unit step function,
- $t = \tan \omega$,
- $G(u, v, t)$, the Fourier transform of $\tilde{f}(x_D, y_D, \omega)$

$f(x, y, z)$ is recovered by three-dimensional inverse Fourier transform
Hence discarding attenuation, under the following conditions:

- first order scattering events are accounted for since they are vastly dominant and higher order scattering are neglected,
- the electron density is assumed to be constant. This is a reasonable hypothesis since most human tissues (brain cells, blood, muscles, lung tissues, water, etc.) have an electron density around $3.4 \times 10^{23}$ cm$^{-3}$. Their density is also around $1.0$ g.cm$^{-3}$ This means for our purpose, objects containing bones should not be considered,
- the set of compounded conical projections has one fixed direction $\textbf{n}$, parallel to the $Oz$ axis direction,
- the set of detecting pixels are distributed as array on a two-dimensional area, forming a collimated SPECT gamma camera.

- Reconstruction is analytically possible
- Data is collected by non-rotating Gamma camera

One may say that the usual spatial rotation angle is replaced by the scattering angle in this modality

Realistic working factors (attenuation, noise, radiation spreading, etc.) may be taken into account in later steps
3D Reconstruction results

Original object (cylinder) in a cube consisting of 16 transaxial planes.
Series of images parameterized by the angle of Compton scattering

\( \omega \ (5^0 < \omega < 175^0) \)
Reconstructed object in the absence of noise (RMSE = 1.2%)
the cone is now a V-line

Compounded V-Line projection

Collimated detecting pixel  Linear gamma camera
Image formation modeling:

Compounded V-Line Radon Transform (CVLRT) of \( f(x, y) \)

\[
\hat{f}(\zeta, \tau) = K^*(\tau) \int_0^\infty \frac{d\eta}{\eta} \int_0^\infty \frac{dr}{r} \left[ f(\zeta + r \sin \omega, \eta + r \cos \omega) + f(\zeta - r \sin \omega, \eta + r \cos \omega) \right]
\]

where
- \( \tau = \tan \omega \),
- \( K^*(\omega) = \pi r^2 \rho^* \mathcal{P}(\omega) \) (Klein-Nishina)
- \( 1/\eta \) and \( 1/r \) (Photometric factors in 2D)

Inversion formula

\[
f(x, y) = \int_\mathbb{R} dq \, e^{2i\pi qy} \int_\mathbb{R} dz \, e^{-2i\pi zq} \frac{h(x, z)}{(-) \left[ \log 2\pi |q| + \gamma - i \frac{\pi}{2} \text{sgn} q \right]}
\]

under the same conditions as in 3D with

\[
h(x, y) = \frac{y}{\pi} \int_0^\pi d\tau \left( \text{P.V.} \int_\mathbb{R} d\zeta \left( \frac{1}{\zeta - x - y\tau} + \frac{1}{\zeta - x + y\tau} \right) \frac{\partial \hat{f}(\zeta, \tau)}{\partial \zeta} \right)
\]
2D Reconstruction

Original Thyroid phantom

V-line data of the Thyroid

Reconstruction of Thyroid phantom

(Morvidone M, Nguyen M K, Truong T T, Zaidi H, IJBI -2010)
B) Transmission-Scattering Tomographic Imaging

- Working conditions:
  - Object is illuminated by external source S of known characteristics
  - Detection of scattered photons by point-like detector D without collimator
  - Collected photon flux density at D for given scattering angle $\omega$ comes from all scattering sites M on circular arcs subtending an inscribed angle of $(\pi - \omega)$.

- Because of the geometry of Compton scattering, this photon flux density is proportional to the integral of the object electron density $n(r, \theta)$ on such circular arcs.

- Such integrals depend on two variables: $\omega$ the scattering angle and $\Phi$ the geometric position of the circular arc as measured from a reference direction.

- Up to now, two possible ways to vary the relative positions of the source S and the detector D in the plane lead to two working modalities for Compton Scatter Tomography (CST):

1) Norton’s modality (1995)

- Source S at fixed position
- Detector D moving along an axis passing through S
- Object is situated on one side of the axis SD
- Radiation spreading and attenuation not accounted for

Modeling by Radon transform on circles intersecting a fixed point with equation

\[ r = p \cos(\theta - \phi) \]

\( \hat{n}(p, \phi) \): transform of the electron density \( n(r, \theta) \)

\[ \hat{n}(p, \phi) = \int_{\text{arc } SD} ds \ n(r, \gamma + \phi) \]

Analytic Inversion Formula (Cormack (1964))

\[ n(r, \theta) = \frac{1}{2\pi^2 r} \int_0^{2\pi} d\phi \int_0^\infty dp \frac{\partial \hat{n}(p, \phi)}{\partial p} \frac{1}{r/p - \cos(\theta - \phi)} \]
Simulation results on Radon transform on circles through O

Original Shepp-Logan phantom

Radon data of the SL phantom

Reconstructed SL phantom with
NMAE = 3.6% and NMSE = 0.5%

2) Our modality (Inverse Problems 2010)

- Object inside circle ($\Gamma_p$) centered at O and of radius $p$
- Source S and Detector D on a rotating diameter of ($\Gamma_p$)
- Radiation spreading and attenuation not accounted for

Radon transform on circular arcs subtended by a rotating chord of fixed length: integration of a function $f$ on circular arc of equation

$$r = r(\cos \gamma) = p \left( \sqrt{1 + \tau^2 \cos^2 \gamma} - \tau \cos \gamma \right)$$

$$\hat{n}(p, \phi)$$, the transform of $n(r, \theta)$ is

$$\hat{n}(p, \phi) = \int_{\text{arc } SD} ds \ n(r, \gamma + \phi)$$

where $ds$ is the arc measure and

$$\theta = (\varphi - \gamma) \ , \ \tau = \cot \omega$$
Inversion via mapping back to the Radon transform on circles intersecting the origin O and using this inversion procedure with

- new variable

\[ g = \frac{1}{2} \left( \frac{p}{r} - \frac{r}{p} \right) \]

- new function

\[ N(g, \theta) = \frac{\tau \, n(r, \theta)}{\sqrt{1 + \tau^2}} \]

- Analytic inversion formula

\[
n(r, \theta) = \frac{1}{2\pi^2} \sqrt{1 + \frac{1}{4} \left( \frac{p}{r} - \frac{r}{p} \right)^2} \int_0^{2\pi} d\phi \int_0^\infty d\tau \frac{1}{2\tau} \frac{1}{\left( \frac{p}{r} - \frac{r}{p} \right) - \cos(\theta - \phi)} \frac{\partial}{\partial \tau} \left( \frac{\tau \, \hat{n}(\tau, \phi)}{\sqrt{1 + \tau^2}} \right)\]

23
Simulation results for the NT’s modality

Original Shepp-Logan (SL) phantom

Reconstructed SL phantom in NT’s modality

Contexte

Modèles directs

Inversion analytique

Travaux en cours

Perspectives

TRAC

1

Formation d'images

Étude de nouvelles modalités de CST & TR généralisées

Gaël RIGAUD

24/01/2012

6 / 14
Contexte

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2

Formation d'images

Étude de nouvelles modalités de CST & TR généralisées

Gaël RIGAUD

24/01/2012
3) Bimodal Compton Scatter radiation imaging (2013)

Combining two modalities of scattered radiation imaging

1) Norton’s Compton scatter tomography CST\(_1\)
2) 2D scattered radiation emission tomography CST\(_2\)

Principle of Norton’s modality of Compton scattering tomography

Principle of compounded \(\Psi\)-line Radon transform
Treatment of attenuation

CST_1 Transmission Data: $C_{\rho \varphi}$

CST_2 Emission Data: $\mathcal{V}_{\xi \tau}$

attenuation map

electron density

activity

Concept of a novel bimodal system

- Image formation with attenuation factor

- Inversion of $C$ without attenuation factor

- Inversion of $\mathcal{V}$ without attenuation factor
\[ F^{n+1} = F^n + D_m^{-1} T^{-1} \circ (G - T^\Phi F^n) \]

with

- \( F^0 \equiv 0 \)
- \( T \equiv C \) or \( V \)
- \( G = C^{\Phi} n_e \) or \( V^{\Phi} a \)
- \( D_m = \text{maximum of the distortion kernel} \)

**Prior information**: we know the kind of matters inside the studied object.

**K-means is performed to estimate attenuation map at each step from the estimate of the electron density.**
Simulation Results

Zubal phantom

CST₁

CST₂

electron density

activity distribution
<table>
<thead>
<tr>
<th>Original</th>
<th>Without noise</th>
<th>30dB SNR</th>
<th>20dB SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron density map</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>attenuation map</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>activity map</td>
<td>![Image]</td>
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Different reconstructions of the electron density map, attenuation map for different SNR
Conclusions

Applications
- Non-destructive testing
- Biomedical imaging

Advantages
- Estimation of $n_e$, $\mu$ and $\alpha$
- Reconstruction without rotation
- Efficient and generalized correction algorithm
CONCLUSION

• New imaging concepts exploiting scattered radiation

• New imaging modalities (emission, transmission, bimodality)

• Mathematical modelling by generalized Radon Transforms (RT) (V-line, Conical, circular arcs RT)

**Advantage:** New operating scheme without detector rotation (planar detector in 3D and linear detector in 2D) in emission imaging for activity reconstruction

**Advantage:** New tomographies using point source – point detector in relative motion on a curve (line or circle) for direct electron density reconstruction
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Thank you for your attention